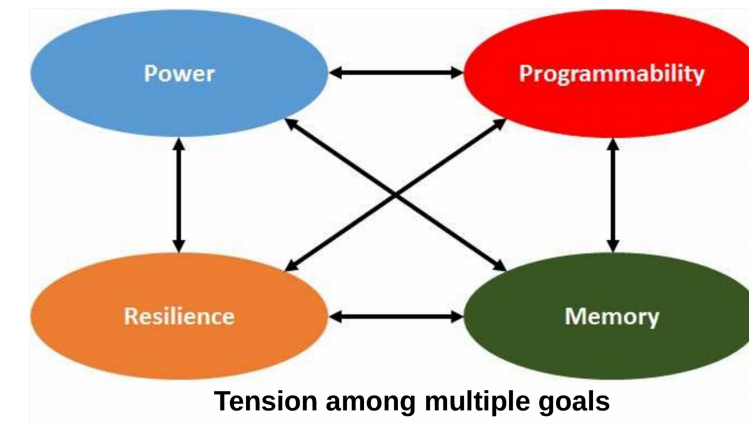


## Motivation

The PyDAC runtime for a FPGA-based heterogeneous architecture demonstrated that algorithms using the Divide-and-Conquer design pattern can balance multiple --- sometimes conflicting --- system goals.



### Key Question:

Can the same success be extended to another class of applications that use, for example, the Wavefront design pattern?

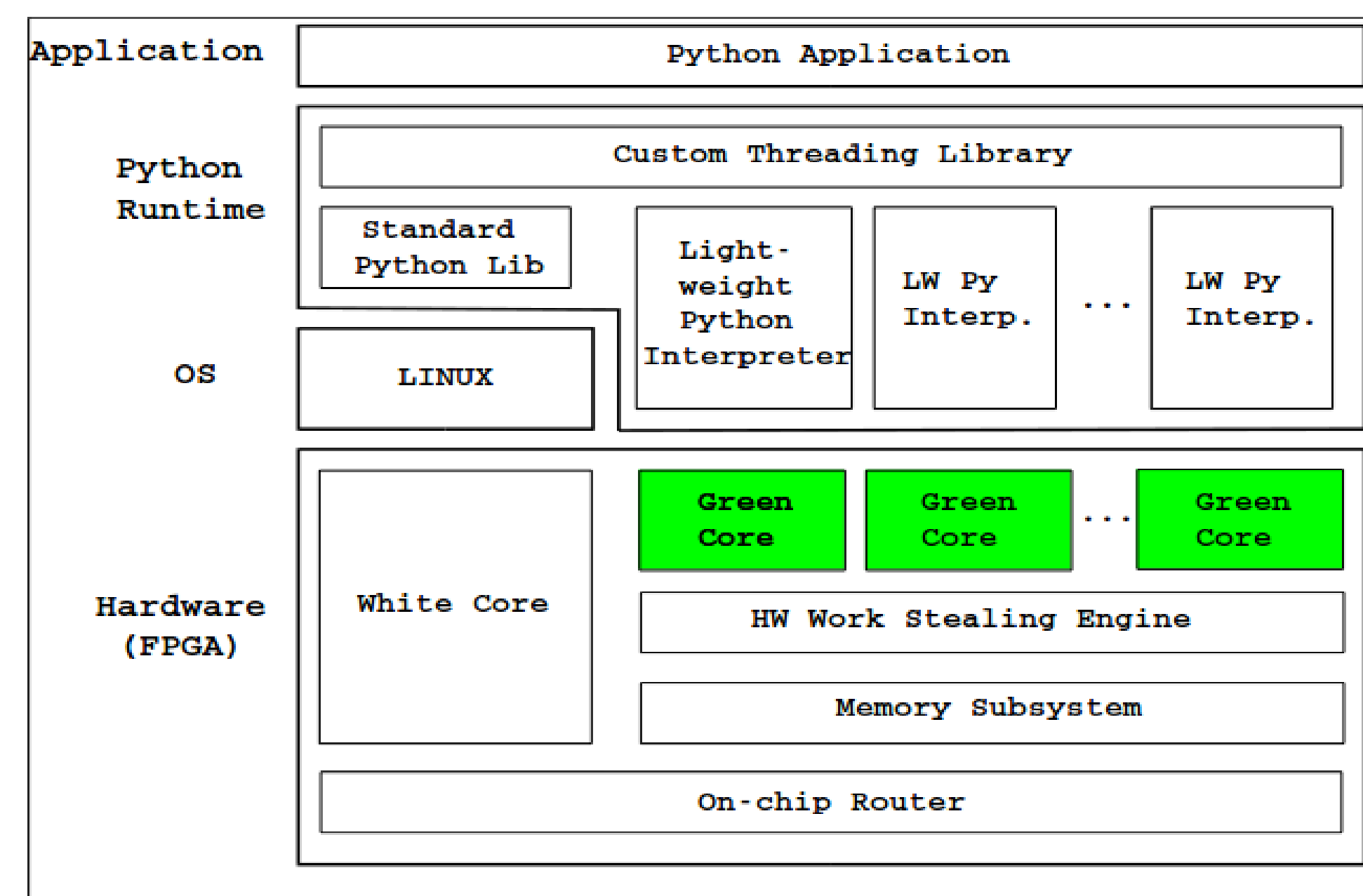
To test this, we left the hardware unchanged and investigated four well-known wavefront algorithms: LU decomposition, QR decomposition, Cholesky decomposition and matrix-matrix multiplication.

Success is defined as:

- ◆ A simple application/runtime interface
- ◆ Efficient use of the memory subsystem

(The hardware does not change so power and resilience are not impacted by these tests.)

## FPGA Heterogeneous Architecture



- ◆ **Match** and **Post** method for memory access
- ◆ Object store and no linear array of memory

Currently implemented in software but part of heterogeneous hardware

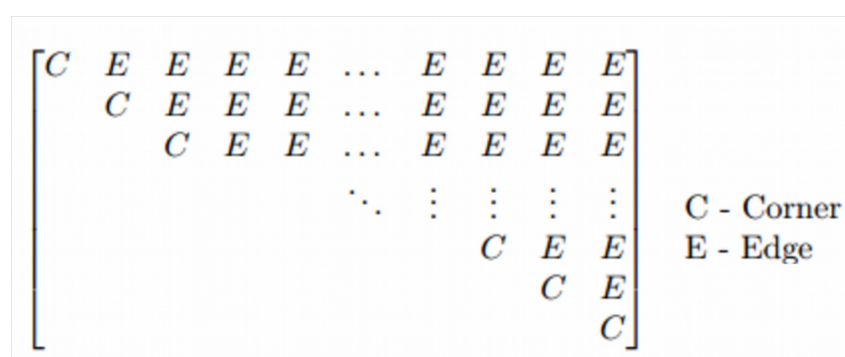
## Examples

```

1 class dpbtrf(waverun):
2     def com_corner(self, Ainput, Sdatajk, Sdatakk):
3         return Ainput - sum([(p**2*q) for p, q in zip(Sdatajk, Sdatakk)])
4     def com_edge(self, Ainput, Sdatajk, Sdatakk, Sdataik, chofact):
5         return Ainput - sum([(p**2*q*r) for p, q, r in zip(Sdatajk, Sdatakk, Sdataik)]) / chofact
6
7
8     def Corner(self, index, step):
9         Res = self.com_corner(Ainput, jkdata, kkdata) #Compute
10        self.dptrb_post(index, step, Res) #Post data
11        return Res
12
13    def Edge(self, index, step):
14        Res = self.com_edge(Ainput, jkdata, kkdata, ikdata, chofact) #Compute
15        self.dptrb_post(index, step, Res) #Post data
16        return Res
17
18    def getFunc(self):
19        Func = ('00': self.Cornet, '0*': self.Edge)
20        return Func
21
22    def primer(self):
23        for i in range(self.data.shape[0] / block):
24            for j in range(self.data.shape[1] / block):
25                if (i < j):
26                    self.post(self.data[i][j], 'Ainput', [i+1, j+1], 0)
27                if (i == 0):
28                    self.post(self.data.shape[0] - j - 1, 'maxi', [i+1, j+1], 0)
29                else:
30                    self.post(-1, 'maxi', [i+1, j+1], 0)
31                    self.post(0, 'Result', [i+1, j+1], 0)

```

Figure 2: Cholesky Decomposition



### Computation:

- Corner  

$$D_{jj} = A_{jj} - \sum_{k=1}^{j-1} U_{jk}^2 D_{kk}$$
- Edge  

$$U_{ij} = \frac{A_{ij} - \sum_{k=1}^{j-1} U_{ik} U_{jk} D_{kk}}{D_{jj}}$$

```

1 class dgemm(waverun):
2     def Mul(self, Ainput, Binput, Cold):
3         return np.add(np.dot(Ainput, Binput), Cold)
4
5     def Corner(self, index, step):
6         #Match data
7         Cnew = self.Mul(Ainput, Binput, Cold)
8         #Post results
9         return Cnew
10
11    def Xedge(self, index, step):
12        #Match data
13        Cnew = self.Mul(Ainput, Binput, Cold)
14        #Post results
15        return Cnew
16
17    def Yedge(self, index, step):
18        #Match data
19        Cnew = self.Mul(Ainput, Binput, Cold)
20        #Post results
21        return Cnew
22
23    def Interior(self, index, step):
24        #Match data
25        Cnew = self.Mul(Ainput, Binput, Cold)
26        #Post results
27        return Cnew
28
29    def getFunc(self):
30        Func = ('00': self.Cornet, '*0': self.Xedge, '0*': self.Yedge, '**': self.Interior)
31        return Func
32
33    def primer(self):
34        for i in range(self.data[0].shape[0] / block):
35            for j in range(self.data[1].shape[1] / block):
36                #Post input data

```

Figure 1: Matrix Multiplication

## Runtime

- ◆ Provides simple user interface
- ◆ **Schedule** and **Gather** methods to schedule the task and gather the result
- ◆ One program for different types of cores
- ◆ Manages memory keys

### Programmer responsibility:

- ◆ Simple object-oriented interface to runtime
- ◆ Provide function based on the index
- ◆ Block of input data via Python method

## Results

A simple application/runtime interface with two user methods is implemented

### Cholesky Decomposition

```

1 Amat = np.random.randint(num, size=(row, column))
2 myapp = dpbtrf(Amat, block)
3 myapp.schedule(row/block)
4 myapp.gather('Result')

```

### Matrix Multiplication

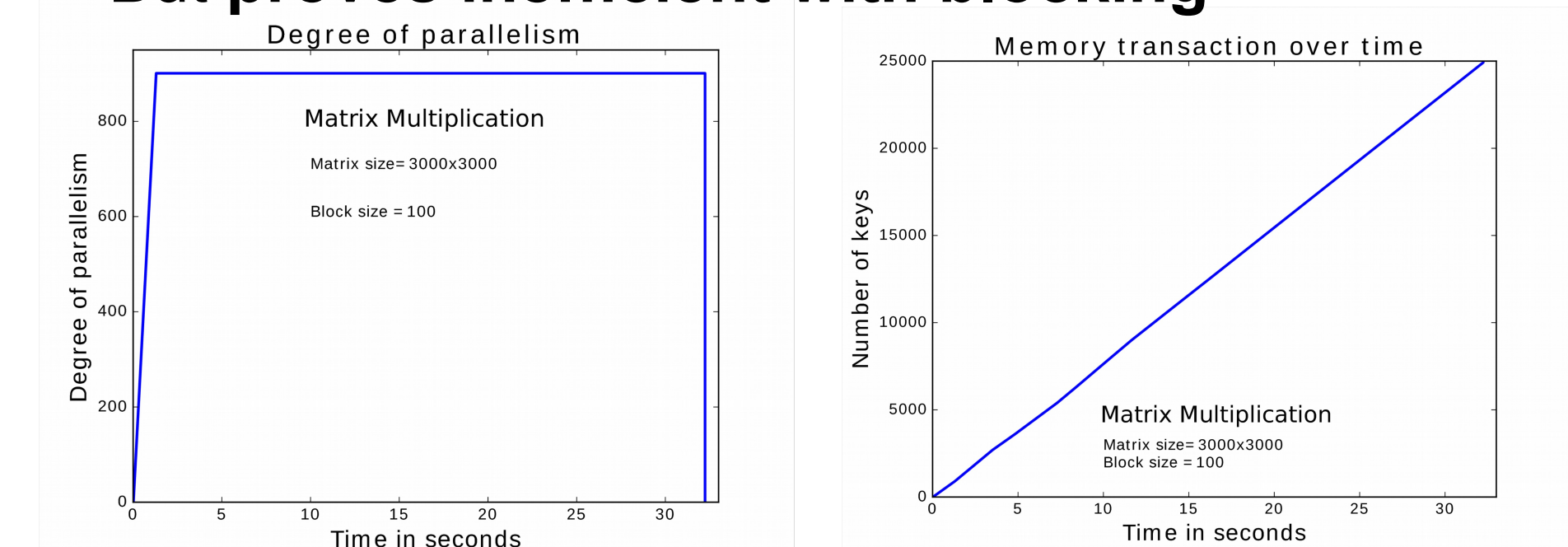
```

1 Amat = np.random.randint(num, size=(row, column))
2 Bmat = np.random.randint(num, size=(row, column))
3 myapp = dgemm(Amat, Bmat, block)
4 myapp.schedule(row/block)
5 myapp.gather('Result')

```

User application using the runtime interface

- ◆ Cholesky, LU, QR, and Matrix multiplication algorithms were implemented using “wavefront” design pattern
- ◆ Cholesky, LU, and QR works well with point-point computation
- ◆ But proves inefficient with blocking



Results of matrix multiplication with blocking

Hardware memory subsystem is tested for transaction bandwidth of range 20k-40k. The runtime efficiently uses the system by generating keys of similar range.

## Background

### Pydac previously implemented:

- ◆ Divide and conquer
- ◆ Proven to be efficient for certain algorithms

### Design Patterns

- ◆ Intel TBB (Parallel patterns)
- ◆ Divide and Conquer
- ◆ Wavefront
- ◆ Agglomeration
- ◆ Elementwise
- ◆ Reduction

B. Huang, R. Sass, N. Debardeleben and S. Blanchard, "Harnessing Unreliable Cores in Heterogeneous Architecture: The PyDac Programming Model and Runtime," 2014 44th Annual IEEE/IFIP International Conference on Dependable Systems and Networks, Atlanta, GA, 2014, pp. 744-749.

## Future Work

- ◆ Developing graph related algorithms for waverun
- ◆ Testing the runtime with the heterogeneous hardware